CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

1348 FURTHER MATHEMATICS

1348/01 Paper 1 (Further Pure Mathematics), maximum raw mark 120

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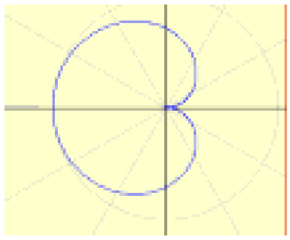
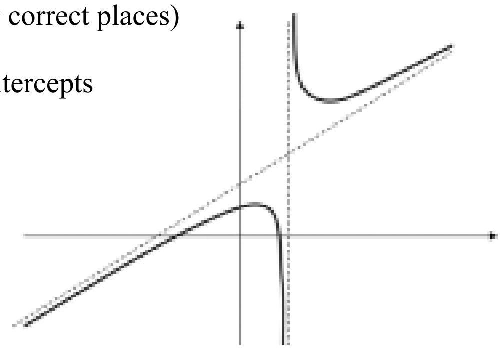
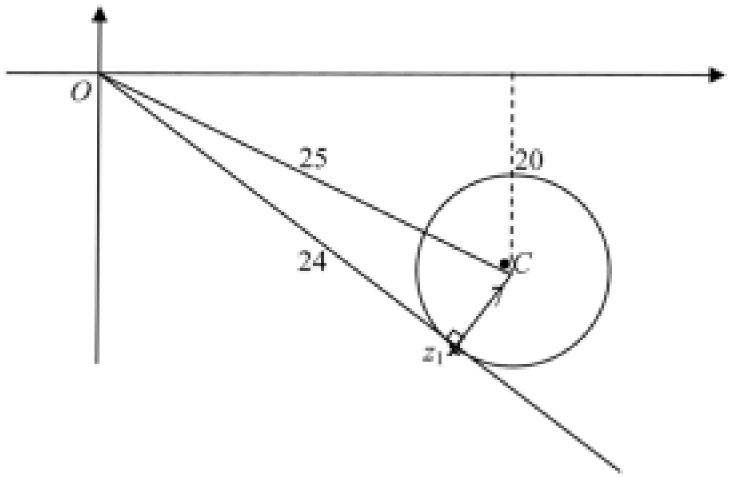
1	use of formula $V = \frac{1}{6}\left|\mathbf{a}\bullet\mathbf{b}\times\mathbf{c}\right|$ or equivalent *NB* $\mathbf{b}\times\mathbf{c} = -4\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$	M1
	attempt at relevant scalar triple product = $\begin{vmatrix} 2 & 3 & -2 \\ 2 & 0 & 4 \\ 6 & 1 & 7 \end{vmatrix} = 18$	M1
	(or a scalar and a vector product)	
	$V = 3$ **cso**	A1 [3]
2	$a_4 = \dfrac{y^{(4)}(1)}{4!} = \dfrac{(-2)^3}{1.3.5.7}$ from the Taylor series expansion	M1 A1
	$\Rightarrow y^{(4)}(1) = \dfrac{(-2)^3}{1.3.5.7} \times 4! = -\dfrac{64}{35}$	A1
	ALTERNATIVE	
	Diff[te]. four times to get $\dfrac{d^4y}{dx^4} = \sum_{n=4}^{\infty} \dfrac{(-2)^{n-1}n(n-1)(n-2)(n-3)(x-1)^{n-4}}{1.3.5....(2n-1)}$	M1 A1
	When $x = 1$, this is $\dfrac{(-2)^3 \times 4(3)(2)(1)}{1.3.5.7} + 0 = -\dfrac{64}{35}$	A1 [3]
3	$n = 1$, $\mathbf{M}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}3\\4\\5\end{pmatrix} = \begin{pmatrix}2(1)+1\\2(1)^2+2(1)\\2(1)^2+2(1)+1\end{pmatrix} \Rightarrow$ result true for $n = 1$ (both sides shown)	B1
	induction hypothesis that $\mathbf{M}^k\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}2k+1\\2k^2+2k\\2k^2+2k+1\end{pmatrix}$	M1
	attempt at $\mathbf{M}^{k+1}\begin{pmatrix}1\\0\\1\end{pmatrix} = \mathbf{M}\begin{pmatrix}2k+1\\2k^2+2k\\2k^2+2k+1\end{pmatrix} = \begin{pmatrix}2k+1-4k^2-4k+4k^2+4k+2\\4k+2-2k^2-2k+4k^2+4k+2\\4k+2-4k^2-4k+6k^2+6k+3\end{pmatrix}$	M1
	$= \begin{pmatrix}2k+3\\2k^2+6k+4\\2k^2+6k+5\end{pmatrix}$	A1
	$= \begin{pmatrix}2(k+1)+1\\2(k+1)^2+2(k+1)\\2(k+1)^2+2(k+1)+1\end{pmatrix}$	A1
	This is the result with k replaced by $(k + 1)$. Hence IF the result is true for $n = k$ THEN it also true for $n = k + 1$. Since the result is true for $n = 1$, it follows that it is true for all positive integers n.	E1 [6]

4	**(i)**	Closed curve containing the pole, O Cusp at the pole Essentially all correct, including at least $(r, \theta) = (1, \pi)$ 	B1 B1 B1 [3]
	(ii)	$A = \left(\frac{1}{2}\right)\int_0^{2\pi}\sin^2\frac{1}{2}\theta\,\mathrm{d}\theta$ Condone missing $\frac{1}{2}$ until final A1	M1
		use of double-angle identity $A = \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{2}-\frac{1}{2}\cos\theta\right)\mathrm{d}\theta$	M1
		correct integration. $= \frac{1}{4}[\theta - \sin\theta]$	A1
		$= \frac{1}{2}\pi$	A1 [4]
5	**(i)**	VA $x = 3$	B1
		$y = \frac{2x^2+5x-25}{x-3} = \frac{2x(x-3)+11(x-3)\ \ [+8]}{x-3}$	M1
		$y = 2x + 11$ oblique asymptote	A1 [3]
	(ii)	for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-3)(4x+5)-\left(2x^2+5x-25\right)}{(x-3)^2}$ (good attempt at the *Quotient Rule*)	M1
		correct unsimplified **OR** $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x+11+\frac{8}{x-3}\right) = 2-\frac{8}{(x-3)^2}$	A1
		solving quadratic equation $4x^2 - 7x - 15 = 2x^2 + 5x - 25$ i.e. $2x^2 - 12x + 10 = 0$ **OR** $(x-3)^2 = 4$	M1
		(1, 9) and (5, 25) (Give one A1 for both x's correct without either y)	A1 A1 [5]
	(iii)	General shape (with asymptotes and turning points in approximately correct places)	B1
		$2x^2 + 5x - 25 = 0$ solved to find x-intercepts	M1
		$x = -5$ or $2\frac{1}{2}$ 	A1 [3]

6	(i)	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$ and $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta \Rightarrow z^n + \frac{1}{z^n} \equiv 2\cos n\theta$	B1 [1]
	(ii)	$\left(z+\frac{1}{z}\right)^5 \equiv \left(z^5+\frac{1}{z^5}\right)+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)$	M1 A1
		Repeated use of result $\Rightarrow 32\cos^5\theta \equiv 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
		$\Rightarrow 16\cos^5\theta \equiv \cos 5\theta + 5\cos 3\theta + 10\cos\theta$ **Answer Given**	A1 [4]
	(iii)	$16\cos^5\theta = \pm\cos\theta$ Condone sign error here to allow for first A1	M1
		$\cos\theta = 0 \Rightarrow \theta = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ (both)	A1
		$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$ or $\frac{5}{3}\pi$ (both)	A1
		$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2}{3}\pi$ or $\frac{4}{3}\pi$ (both)	A1
		but allow one A1 for $\theta = \frac{1}{3}\pi$ and $\theta = \frac{2}{3}\pi$ if both 2nd answers missing	[4]
7	(i)	Circle with centre (20, –15)	B1
		Circle of radius = 7 stated or deducible from diagram	B1
		Allow B1 for a circle entirely in the 4th Quad.	
		Interior of circle shaded (Ignore boundary in/out)	B1 [3]
	(ii)		
		for z_1 in correct place	B1
		for (sufficient) distances	B1
		for $\arg(z_1) = -\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{7}{24}\right)$ or equivalent, using other inverse trig. functions	M1
		[NB – this is $\left(-\tan^{-1}\frac{4}{3}\right)$ using a result in Q13]	
		for – 0.927 (or 2π – 0.927 = 5.356)	A1 [4]

8	(i)	(group table below)	B4, 3, 2, 1, 0 [4]
	(ii)	Subgroups of order 2: $\{e, a\}$, $\{e, b\}$, $\{e, c\}$, $\{e, ab\}$, $\{e, bc\}$, $\{e, ca\}$, $\{e, abc\}$ 6 distinct given all 7, no repeats Subgroups of order 4: $\{e, a, b, ab\}$, $\{e, b, c, bc\}$, $\{e, c, a, ca\}$ $\{e, a, bc, abc\}$, $\{e, b, ca, abc\}$, $\{e, c, ab, abc\}$ $\{e, ab, bc, ca\}$ **Either** (all 3 of 1st type) + **B1** (all3 of 2nd type) + **B1** (last one) **Or** (5 distinct) + **B1** (6th) + **B1** (all 7 and no repeats)	B1 B1 B2 [5]
9	(i)	Comp. Fn. is $u = A\cos 2x + B\sin 2x$ Allow $Ae^{2ix} + Be^{-2ix}$ here For Part. Int. try $u = ax + b$ ($u' = a$ and $u'' = 0$) and substd. into given d.e. $a = 2$, $b = \frac{1}{4}$ Gen. Soln. is $u = A\cos 2x + B\sin 2x + 2x + \frac{1}{4}$ **ft** Must have two arbitrary constants Condone apparently complex coefficients here Don't allow $Ae^{2ix} + Be^{-2ix}$ here	M1 A1 M1 A1 B1 [5]
	(ii)	$xy = u \Rightarrow x\frac{dy}{dx} + y = \frac{du}{dx}$ and $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \frac{d^2u}{dx^2}$ $\Rightarrow$ (*) becomes $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy = 8x + 1$ $\Rightarrow y = \frac{A\cos 2x}{x} + \frac{B\sin 2x}{x} + 2 + \frac{1}{4x}$ **ft** Accept $xy = \ldots$ Must have two real arbitrary constants	M1 A1 A1 B1 [4]

Group table for 8 (i):

	e	*a*	*b*	*c*	*ab*	*bc*	*ca*	*abc*
e	*e*	*a*	*b*	*c*	*ab*	*bc*	*ca*	*abc*
a	*a*	*e*	*ab*	*ca*	*b*	*abc*	*c*	*bc*
b	*b*	*ab*	*e*	*bc*	*a*	*c*	*abc*	*ca*
c	*c*	*ca*	*bc*	*e*	*abc*	*b*	*a*	*ab*
ab	*ab*	*b*	*a*	*abc*	*e*	*ca*	*bc*	*c*
bc	*bc*	*abc*	*c*	*b*	*ca*	*e*	*ab*	*a*
ca	*ca*	*c*	*abc*	*a*	*bc*	*ab*	*e*	*b*
abc	*abc*	*bc*	*ca*	*ab*	*c*	*a*	*b*	*e*

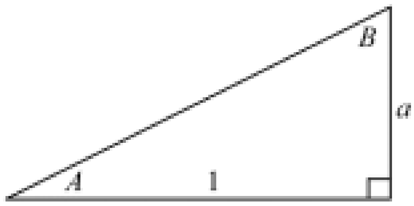
10	(i)	d.v. = $\begin{pmatrix}1\\7\\-6\end{pmatrix}\times\begin{pmatrix}3\\-5\\8\end{pmatrix}=\begin{pmatrix}26\\-26\\-26\end{pmatrix}\equiv\begin{pmatrix}-1\\1\\1\end{pmatrix}$	M1 A1
		finding one point on line: e.g. (0, 8, 11), (8, 0, 3), (11, –3, 0)	M1 A1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …, ***l*** = …)	B1
			[5]
		ALTERNATIVE 1	
		finding two points on line: e.g. (0, 8, 11), (1, 7, 10), (8, 0, 3), (11, –3, 0) , …	M1 A1 A1
		ft d.v. $\mathbf{r}=\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (e.g.)	B1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …)	B1
			[5]
		ALTERNATIVE 2	
		Eliminating x (say): $z = y + 3$	M1 A1
		Setting $y = \lambda$ (or equivalent) and finding z and x in terms of the parameter	M1
		$z = \lambda + 3$ and $x = 8 - \lambda$	A1
		ft Line equation in vector form $\mathbf{r}=\begin{pmatrix}8\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (must have **r** = …)	B1
			[5]

(ii)	$\begin{vmatrix} 1 & 7 & -6 \\ 3 & -5 & 8 \\ k & 2 & 3 \end{vmatrix} = 26k - 130$	M1 A1
	$= 0$ when $k = 5$ **ft** from a linear eqn.	A1
	e.g. $\begin{matrix} x + 7y - 6z = -10 \\ 3x - 5y + 8z = 48 \\ 5x + 2y + 3z = 16 \end{matrix} \rightarrow \begin{matrix} -33y + 33z = 66 \\ -26y + 26z = 78 \end{matrix} \rightarrow \begin{matrix} -y + z = 2 \\ -y + z = 3 \end{matrix}$	
	eliminating one variable (twice); correctly Inconsistency noted/explained (allow valid **ft**)	M1 A1 A1 [5]
	ALTERNATIVE	
	Substituting $x = 8 - \lambda$, $y = \lambda$ and $z = 3 + \lambda$ into $kx + 2y + 3z = 16$ $\Rightarrow \lambda(5 - k) + (11k - 22) = 0$ λ terms; other terms $k = 5$ gives non-unique soln. Then $33 = 0$ **ft** and system inconsistent (allow valid **ft**)	M1 A1 A1 A1 A1 B1 [5]
11 (a)	Roots $\{qr, rp, pq\} = \left\{\frac{pqr}{p}, \frac{pqr}{q}, \frac{pqr}{r}\right\}$	M1
	$= \left\{\frac{4}{p}, \frac{4}{q}, \frac{4}{r}\right\}$ from $pqr = 4$, so subst. $y = \frac{4}{x}$	A1
	Setting $x = \frac{4}{y} \Rightarrow \frac{64}{y^3} + 2.\frac{16}{y^2} + 3.\frac{4}{y} - 4 = 0$	M1
	$\Rightarrow 64 + 32y + 12y^2 - 4y^3 = 0$	M1 A1
	$\Rightarrow y^3 - 3y^2 - 8y - 16 = 0$	A1 [6]
	ALTERNATIVE (for last four-mark section)	
	$\sum \alpha' = qr + rp + pq = 3$	B1
	$\sum \alpha'\beta' = pqr(p + q + r) = 4 \times (-2) = -8$	B1
	$\sum \alpha'\beta'\gamma' = (pqr)^2 = 16$	B1
	New eqn. is $x^3 - 3x^2 - 8x - 16 = 0$ (must have "= 0")	B1 [6]
(b) (i)	β and γ are complex conjugates (since the coefficients are real)	B1 [1]
(ii)	$\alpha\beta\gamma = 4 \Rightarrow \beta\gamma = \frac{4}{\alpha}$	
	and $\lvert\beta\rvert = \lvert\gamma\rvert$ since conjugates $\Rightarrow \lvert\beta\gamma\rvert = \lvert\beta\rvert.\lvert\gamma\rvert = \lvert\beta\rvert^2 = \frac{4}{\alpha}$	M1

	and $\lvert\beta\rvert = \dfrac{2}{\sqrt{\alpha}}$ (since $\alpha > 0$ given) **Answer Given**	A1	[2]
(iii)	p(2.695) = –0.003… < 0 and p(2.705) = 0.049… > 0 so 2.695 < α < 2.705 (by the "*Change-of-Sign Rule*") $\Rightarrow$ α = 2.70 to 3s.f. (Allow numerical methods, e.g. *Newton-Raphson* – to find α = 2.70 to 3s.f.)	B1	
	$\alpha + \beta + \gamma = 4 \Rightarrow \beta + \gamma = 4 - \alpha$ Then 2.695 < α < 2.705 $\Rightarrow$ 1.295 < 4 – α < 1.305	M1	
	Now, if $\beta = u + \mathrm{i}v$, then $\gamma = u - \mathrm{i}v$ and $\beta + \gamma = 2u$, i.e. 2 Re(β) Thus 1.295 < 2 Re(β) < 1.305 $\Rightarrow$ 0.6475 < Re(β) < 0.6525	M1	
	and Re(β) = 0.65 to 2 s.f. (A0 for failure to justify 2s.f. accuracy properly)	A1	[4]

12	**(i)**	**(a)** $I_1 = \int_0^2 x\sqrt{1+2x^2}\,\mathrm{d}x = \left[\tfrac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}\right]_0^2 = \tfrac{13}{3}$	M1 A1 A1 [3]
		(b) $I_n = \int_0^2 x^{n-1}.\,x\sqrt{1+2x^2}\,\mathrm{d}x$ Correct splitting and use of parts	M1
		$= \left[x^{n-1}.\,\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}\right]_0^2 - \int_0^2 (n-1)x^{n-2}.\,\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}\,\mathrm{d}x$	A1 A1
		$= 2^{n-1}\,.\,\frac{27}{6} - 0 \qquad - \frac{1}{6}(n-1)\int_0^2 x^{n-2}.\,\left(1+2x^2\right)\sqrt{1+2x^2}\,\mathrm{d}x$	M1
		$= 2^{n-1}\,.\,\frac{27}{6} - \frac{1}{6}(n-1)\left\{2I_n + I_{n-2}\right\}$	A1
		$\Rightarrow 6\,I_n = 27\times 2^{n-1} - 2(n-1)I_n - (n-1)I_{n-2}$	
		$\Rightarrow (2n+4)\,I_n = 27\times 2^{n-1} - (n-1)I_{n-2}$ **Answer Given**	A1 [6]
		(c) $I_0 = \int_0^2 \sqrt{1+2x^2}\,\mathrm{d}x$	
		Let $x\sqrt{2} = \sinh\theta$ $\quad$ ($\sqrt{2}\,\mathrm{d}x = \cosh\theta\,\mathrm{d}\theta$, $\sqrt{1+2x^2} = \cosh\theta$)	M1
		full substitution $I_0 = \frac{1}{\sqrt{2}}\int \cosh^2\theta\,\mathrm{d}\theta$ $\quad$ (Ignore limits for now)	M1 A1
		trig. identity $= \frac{1}{2\sqrt{2}}\int\left(1+\cosh 2\theta\right)\mathrm{d}\theta$	M1
		$= \frac{1}{2\sqrt{2}}\left[\theta + \frac{1}{2}\sinh 2\theta\right]$ **ft** ∫n. of $a + b\cosh 2\theta$ only	A1
		$(0, 2) \rightarrow (0, \sinh^{-1} 2\sqrt{2})$ Limits properly dealt with	M1
		$= \frac{1}{2\sqrt{2}}\left[\sinh^{-1} 2\sqrt{2} + 6\sqrt{2}\right]$	
		$\sinh^{-1} 2\sqrt{2} = \ln\left(3+2\sqrt{2}\right)$ to at least here $= \ln\left(1+\sqrt{2}\right)^2 = 2\ln\left(1+\sqrt{2}\right)$	M1
		$I_0 = 3 + \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ legitimately **Answer Given**	A1 [8]

	ALTERNATIVE (partial)	
	Let $x\sqrt{2} = \tan\theta$ $\left(\sqrt{2}\mathrm{d}x = \sec^2\theta\,\mathrm{d}\theta\right)$	M1
	full substitution $I_0 = \frac{1}{\sqrt{2}}\int \sec^3\theta\,\mathrm{d}\theta$ (Ignore limits for now)	M1 A1
	use of ∫n. by parts on $\int \sec\theta\sec^2\theta\,\mathrm{d}\theta$ and use of $\tan^2\theta = \sec^2\theta - 1$	M1
	$2\sqrt{2}I_0 = \sec\theta\tan\theta + \ln\|\sec\theta + \tan\theta\|$	A1
	Further progress (limits, etc.) essentially impossible	
	ALTERNATIVE	
	$I_0 = \int_0^2 \sqrt{1+2x^2} \times 1\,\mathrm{d}x = x\sqrt{1+2x^2} - \int \tfrac{1}{2}\left(1+2x^2\right)^{-\frac{1}{2}} . 4x . x\,\mathrm{d}x$ ∫n. by parts	M1 A1
	$= x\sqrt{1+2x^2} - \int \frac{\left(2x^2+1\right)-1}{\sqrt{1+2x^2}}\,\mathrm{d}x$	M1
	$= x\sqrt{1+2x^2} - I_0 + \int \frac{1}{\sqrt{1+2x^2}}\,\mathrm{d}x$	M1
	$\Rightarrow 2I_0 = x\sqrt{1+2x^2} + \int \frac{1}{\sqrt{1+2x^2}}\,\mathrm{d}x$	
	$\Rightarrow I_0 = \frac{1}{2}\left(x\sqrt{1+2x^2} + \frac{1}{\sqrt{2}}\sinh^{-1}\left(x\sqrt{2}\right)\right)$ (from MF20)	M1 A1
	Use of limits (0, 2) $\Rightarrow I_0 = 3 + \frac{1}{2\sqrt{2}}\sinh^{-1}(2\sqrt{2})$	
	$\sinh^{-1} 2\sqrt{2} = \ln\left(3+2\sqrt{2}\right) = \ln\left(1+\sqrt{2}\right)^2 = 2\ln\left(1+\sqrt{2}\right)$	M1
	$I_0 = 3 + \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ legitimately **Answer Given**	A1 [8]
(ii)	$y = \frac{1}{\sqrt{2}}x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x\sqrt{2}$ and $S = 2\pi\int_0^2 \frac{x^2}{\sqrt{2}}.\sqrt{1+2x^2}\,\mathrm{d}x$	M1
	$= \pi\sqrt{2}\left(I_2\right)$	A1
	use of R.F. for $n = 2$: $8I_2 = 54 - I_0$	M1
	use of *their* **(i)** result $= 51 - \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$	M1
	$\Rightarrow S = \frac{\pi\sqrt{2}}{8}\left(51 - \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)\right)$ or $\frac{\pi}{8}\left(51\sqrt{2} - \ln\left(1+\sqrt{2}\right)\right)$	A1 [5]

13	(i)	 $\tan A = \frac{a}{1}$, $\tan B = \frac{1}{a}$ $\Rightarrow \tan^{-1} a + \tan^{-1}\frac{1}{a} = A + B = \frac{\pi}{2}$	B1 [1]
	(ii)	$\tan\left(\tan^{-1} a \pm \tan^{-1} b\right) = \frac{\tan\left(\tan^{-1} a\right) \pm \tan\left(\tan^{-1} b\right)}{1 \mp \tan\left(\tan^{-1} a\right).\tan\left(\tan^{-1} b\right)} = \frac{a \pm b}{1 \mp ab}$	M1 A1 [2]
	(iii)	$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage	M1 A1
		$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \tan^{-1}(2) + \tan^{-1}\left(\frac{1}{2}\right) + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$ Splitting off 1st 2 terms	
		$= \frac{\pi}{2} + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$ using (i)'s result	B1
		use of difference method (finite or infinite series)	M1
		$\Rightarrow \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \frac{\pi}{2} +$ $\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7}...\right)$ $-\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7} + ...\right)$	
		Cancelling of terms made clear	M1
		$= \frac{\pi}{2} + \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$	A1
		and $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$ using (ii)'s result	B1
		leading to … **Answer Given**	[7]

	ALTERNATIVE	
	$\tan^{-1}\left(\frac{1}{n-1}\right)-\tan^{-1}\left(\frac{1}{n+1}\right)=\tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage	M1 A1
	Use of difference method (finite or infinite series)	M1
	$\sum_{n=1}^{N}\tan^{-1}\left(\frac{2}{n^2}\right) = \left(\tan^{-1}\frac{1}{0}+\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+...\right)$	
	$-\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+...\right)$	A1
	Cancelling of remaining terms made clear	M1
	$\tan^{-1}\infty+\tan^{-1}1 = \frac{\pi}{2}+\frac{\pi}{4}$	A1
	$=\frac{3\pi}{4}$ **Given Answer**	A1
		[7]